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SOLUTIONS OF PROBLEMS.

ALGEBRA.

434. Proposed by S. A. JOFFE, New York City.

Express the "difference of zero" $\Delta^n 0^{n+1}$ in the form: $c_1(n+2)! - c_2(n+1)!$, where c_1 and c_2 are numerical coefficients independent of n.

Solution by the Proposer.

From the theory of Finite Differences we know that

$$\Delta^n 0^i = n(\Delta^n 0^{i-1} + \Delta^{n-1}0^{i-1}).$$

If we put here $i = n + 1$, we obtain

$$\Delta^n 0^{n+1} = n(\Delta^n 0^n + \Delta^{n-1}0^n),$$

which, after division by $n!$, gives

$$\frac{\Delta^n 0^{n+1}}{n!} = \frac{\Delta^n 0^n}{(n-1)!} + \frac{\Delta^{n-1}0^n}{(n-1)!},$$

or, since $\Delta^n 0^n = n!$,

$$\frac{\Delta^n 0^{n+1}}{n!} - \frac{\Delta^{n-1}0^n}{(n-1)!} = n. \qquad (1)$$

Replacing n in equation (1) successively by $n-1$, $n-2$, $\cdots$, 1, we get the following $n-1$ similar equations:

$$\frac{\Delta^{n-1}0^n}{(n-1)!} - \frac{\Delta^{n-2}0^{n-1}}{(n-2)!} = n-1; \quad \frac{\Delta^{n-2}0^{n-1}}{(n-2)!} - \frac{\Delta^{n-3}0^{n-2}}{(n-3)!} = n-2; \quad \cdots \quad \frac{\Delta^2 0^3}{2} - \frac{\Delta 0^2}{1} = 2; \quad \frac{\Delta 0^2}{1} = 1.$$

Adding together the preceding n equations, including (1), we obtain, after cancellation,

$$\frac{\Delta^n 0^{n+1}}{n!} = n + (n-1) + (n-2) + \cdots + 1 = \frac{n(n+1)}{2},$$

which may be written in this form:

$$\Delta^n 0^{n+1} = \frac{n}{2}(n+1)! \qquad (2)$$

Finally, since $\frac{n}{2} = \frac{1}{2}(n+2) - 1$, equation (2) may be expressed as follows:

$$\Delta^n 0^{n+1} = \frac{1}{2}(n+2)! - (n+1)! \qquad (3)$$

which is the form desired.

435. Proposed by C. N. SCHMALL, New York City.

Show that $(e-1) - \frac{1}{2}(e-1)^2 + \frac{1}{3}(e-1)^3 - \cdots = 1$, where e is the Naperian base of logarithms.

Solution by Geo. W. Hartwell, Hamline University.

This equation results from substituting $(e-1)$ for x in the series,

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots.$$

This, however, is not a permissible substitution, since the logarithmic series converges only when $x \leqq 1$. Moreover, the given series is not convergent, since

$$\operatorname*{Lt}_{n \doteq \infty} \frac{(e-1)^n}{n} \neq 0,$$

$e - 1$ being greater than unity.

Also solved by HORACE OLSON, C. E. FLANAGAN, J. W. CLAWSON, and H. S. UHLER.

GEOMETRY.

449. Proposed by H. E. TREFETHEN, Colby College.

Find a tetrahedron with the face angles at one vertex in arithmetical progression and its six edges expressed in positive integers.

SOLUTION BY THE PROPOSER.

Let the angles be $A + B$, A, $A - B$; the lateral edges x, y, z; the base edges a, b, c; so that $a^2 = x^2 + y^2 - 2xy \cos(A + B)$, $b^2 = x^2 + z^2 - 2xz \cos A$, $c^2 = y^2 + z^2 - 2yz \cos(A - B)$. If, for brevity, we put $1 + \cos(A + B) = P/2$, $1 + \cos A = Q/2$, and $1 + \cos(A - B) = R/2$, we may write

$$a^2 = (x + y)^2 - Pxy = (x + y - p)^2 \text{ or } Pxy - 2py = 2px - p^2, \tag{1}$$

$$b^2 = (x + z)^2 - Qxz = (x + z - q)^2 \quad Qxz - 2qz = 2qx - q^2, \tag{2}$$

$$c^2 = (y + z)^2 - Ryz = (y + z - r)^2 \quad Ryz - 2ry = 2rz - r^2. \tag{3}$$

Eliminating y and z from (1), (2), (3), and arranging we have

$$(2px - p^2)[q(4r - Rq) + 2(Rq - Qr)x] = r(Px - 2p)[2q(r - q) + (4q - Qr)x]. \tag{4}$$

If we put the coefficient of $x^2 = 0$, then

$$p = \frac{Pr(4q - Qr)}{4(Rq - Qr)} \tag{5}$$

and also

$$2x = \frac{p^2q(4r - Rq) + 4pqr(q - r)}{p(8qr - Qr^2 - Rq^2) - p^2(Rq - Qr) + Pqr(q - r)}. \tag{6}$$

We may now assign values to A and B and thus determine P, Q, R. If values are assigned to q and r also, p is defined by (5), and then x may be found from (6), y and z from (1), (2), (3) and finally a, b, c also.

Thus if $A = 90°$ and $B = \text{arc-sin } 1/3$, then $P = 4/3$, $Q = 2$, $R = 8/3$; and if also $q = r = 1$, then $p = 1$, $x = 1/4$, $y = 3/10$, $z = 1/3$. Reducing the values of x, y, z to a common denominator and rejecting it, we have in integers $x = 15$, $y = 18$, $z = 20$, and consequently $a = 27$, $b = 25$, $c = 22$. Or since the equations are symmetrical we may use the reciprocals $x = 4$, $y = 10/3$, $z = 3$, whence in integers $x = 12$, $y = 10$, $z = 9$, and then $a = 18$, $b = 15$, $c = 11$. Again if $\sin B = 1/2$, the angles are 120°, 90°, and 60°; $x = 9$, $y = 15$, $z = 40$; $a = 21$, $b = 41$, $c = 35$. If $\sin B = 2/3$, we find $x = 1{,}092$, $y = 416$, $z = 81$; $a = 2{,}804$, $b = 2{,}185$, $c = 367$.

By varying q, r and the $\sin B$, other sets of numbers may be found ad libitum.

464. Proposed by FRANK R. MORRIS, Glendale, Calif.

The sum of the hypotenuse and one side of a right triangle is 100 feet. A point on the hypotenuse is 10 feet from each of the sides. Find the length of the hypotenuse correct to the third decimal place.

SOLUTION BY W. W. BURTON, Mercer University, Macon, Ga.

Let $BC = x$. Then $AB = 100 - x$ and $BD = x - 10$. The right triangles ACB and PDB are similar. (Their sides are respectively parallel.) Therefore

$$AB : BC = PB : BD \text{ or } (100 - x) : x = PB : (x - 10),$$